Determine whether each trinomial is a perfect square trinomial. Write yes or no. If so, factor it.

1. $25x^2 + 60x + 36$

**SOLUTION:**
The first term is a perfect square.

$25x^2 = (5x)^2$

The last term is a perfect square.

$36 = 6^2$

The middle term is equal to $2ab$.

$60x = 2(5x)(6)$

So, $25x^2 + 60x + 36$ is a perfect square trinomial.

$25x^2 + 60x + 36 = (5x)^2 + 2(5x)(6) + 6^2$

$= (5x + 6)^2$

2. $6x^2 + 30x + 36$

**SOLUTION:**
The first term is not a perfect square. So, $6x^2 + 30x + 36$ is not a perfect square trinomial.
8-9 Perfect Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

3. \(2x^2 - x - 28\)

**SOLUTION:**
The polynomial is not a perfect square or a difference of squares. Try to factor using the general factoring pattern. In this trinomial, \(a = 2, b = -1\) and \(c = -28\), so \(m + p\) is negative and \(mp\) is negative. Therefore, \(m\) and \(p\) must have different signs. List the factors of \(2(-28)\) or \(-56\) with a sum of \(-1\).

<table>
<thead>
<tr>
<th>Factors of (-56)</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, (-56)</td>
<td>(-55)</td>
</tr>
<tr>
<td>(-1, 56)</td>
<td>55</td>
</tr>
<tr>
<td>2, (-28)</td>
<td>(-26)</td>
</tr>
<tr>
<td>(-2, 28)</td>
<td>26</td>
</tr>
<tr>
<td>4, (-14)</td>
<td>(-10)</td>
</tr>
<tr>
<td>(-4, 14)</td>
<td>10</td>
</tr>
<tr>
<td>7, (-8)</td>
<td>(-1)</td>
</tr>
<tr>
<td>(-7, 8)</td>
<td>1</td>
</tr>
</tbody>
</table>

The correct factors are 7 and \(-8\).

\[
2x^2 - x - 28 = 2x^2 - 8x + 7x - 28 \quad m = -8 \text{ and } p = 7
\]

\[
= (2x^2 - 8x) + (7x - 28) \quad \text{Group terms.}
\]

\[
= 2x(x - 4) + 7(x - 4) \quad \text{Factor GCF from each group.}
\]

\[
= (x - 4)(2x + 7) \quad (x - 4) \text{is the common factor.}
\]
8-9 Perfect Squares

4. $6x^2 - 34x + 48$

**SOLUTION:**
Factor the GCF of 2 from each term. 

$$6x^2 - 34x + 48 = 2\left(3x^2 - 17x + 24\right)$$

The resulting polynomial is not a perfect square or a difference of squares. Try to factor using the general factoring pattern.

In the trinomial, $a = 3$, $b = -17$ and $c = 24$, so $m + p$ is negative and $mp$ is negative. Therefore, $m$ and $p$ must have different signs. List the factors of 3(24) or $-60$ with a sum of $-17$.

<table>
<thead>
<tr>
<th>Factors of 72</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1, -72$</td>
<td>$-73$</td>
</tr>
<tr>
<td>$-2, -36$</td>
<td>$-38$</td>
</tr>
<tr>
<td>$-3, -24$</td>
<td>$-27$</td>
</tr>
<tr>
<td>$-4, -18$</td>
<td>$-22$</td>
</tr>
<tr>
<td>$-6, -12$</td>
<td>$-18$</td>
</tr>
<tr>
<td>$-8, -9$</td>
<td>$-17$</td>
</tr>
</tbody>
</table>

The correct factors are $-8$ and $-9$.

$$6x^2 - 34x + 48 = 2\left(3x^2 - 17x + 24\right)$$

$$= 2\left(3x^2 - 9x - 9x + 24\right)$$

$m = -9$ and $p = -8$

$$= 2\left[\left(3x^2 - 9x\right) - \left(9x - 24\right)\right]$$

Group terms with common factors.

$$= 2\left[3x(x - 3) - 8(x - 3)\right]$$

Factor GCF from each group.

$$= 2(x - 3)(3x - 8)$$

$(x - 3)$is the common factor.

5. $4x^2 + 64$

**SOLUTION:**
The polynomial is not a perfect square or a difference of squares. Try to factor the GCF.

The greatest common factor of each term is 4.

$$4x^2 + 64 = 4\left(x^2 + 16\right)$$
6. $4x^2 + 9x - 16$

**SOLUTION:**
The polynomial is not a perfect square or a difference of squares. Try to factor using the general factoring pattern.
In the trinomial, $a = 4$, $b = 9$ and $c = -16$, so $m + p$ is positive and $mp$ is negative. Therefore, $m$ and $p$ must have different signs.
List the factors of $4(-16)$ or $-64$ with a sum of 9.

<table>
<thead>
<tr>
<th>Factors of $-64$</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, $-64$</td>
<td>$-63$</td>
</tr>
<tr>
<td>$-1$, $64$</td>
<td>63</td>
</tr>
<tr>
<td>2, $-32$</td>
<td>$-30$</td>
</tr>
<tr>
<td>$-2$, $32$</td>
<td>30</td>
</tr>
<tr>
<td>4, $-16$</td>
<td>$-14$</td>
</tr>
<tr>
<td>$-4$, 16</td>
<td>14</td>
</tr>
<tr>
<td>4, $-15$</td>
<td>$-11$</td>
</tr>
<tr>
<td>$-8$, 8</td>
<td>11</td>
</tr>
</tbody>
</table>

There are no factors of $-64$ with a sum of 9.
So, this trinomial is prime.

**Solve each equation. Confirm your answers using a graphing calculator.**
7. $4x^2 = 36$

**SOLUTION:**
$4x^2 = 36$
$x^2 = 9$
$x = \pm \sqrt{9}$
$x = \pm 3$
The roots are $-3$ and $3$.
Confirm the roots using a graphing calculator. Let $Y1 = 4x^2$ and $Y2 = 36$. Use the **intersect** option from the **CALC** menu to find the points of intersection.

Thus, the solutions are $-3$ and $3$. 
8-9 Perfect Squares

8. \(25a^2 - 40a = -16\)

**SOLUTION:**
Rewrite with 0 on the right side.

\[
25a^2 - 40a = -16 \\
25a^2 - 40a + 16 = 0
\]

The trinomial is a perfect square trinomial.

\[
25a^2 - 40a = -16 \\
25a^2 - 40a + 16 = 0 \\
(5a)^2 - 2(5a)(4) + (4)^2 = 0 \\
(5a - 4)^2 = 0
\]

\[
5a - 4 = 0 \\
5a = 4 \\
a = \frac{4}{5}
\]

The root is \(\frac{4}{5}\) or 0.8.

Confirm the root using a graphing calculator. Let \(Y_1 = 25a^2 - 40a\) and \(Y_2 = -16\). Use the `intersect` option from the `CALC` menu to find the points of intersection.

Thus the solution is \(\frac{4}{5}\).
8-9 Perfect Squares

9. \(64y^2 - 48y + 18 = 9\)

**SOLUTION:**
Rewrite the trinomial with \(-\) on the right side.

\[
64y^2 - 48y + 18 = 9 \\
64y^2 - 48y + 9 = 0
\]

The resulting trinomial \(64y^2 - 48y + 9 = 0\) is a perfect square trinomial.

\[
\begin{align*}
64y^2 - 48y + 18 &= 9 \\
64y^2 - 48y + 9 &= 0 \\
(8y)^2 - 2(8y)(3) + (3)^2 &= 0 \\
(8y - 3)^2 &= 0 \\
8y - 3 &= 0 \\
8y &= 3 \\
y &= \frac{3}{8}
\end{align*}
\]

The root is \(\frac{3}{8}\) or 0.375.

Confirm the roots using a graphing calculator. Let \(Y1 = 64y^2 - 48y + 18\) and \(Y2 = 9\). Use the **intersect** option from the **CALC** menu to find the points of intersection.

Thus, the solution is \(\frac{3}{8}\). 

---

8. Dimensions of Model B pool could be 20 ft by 50 ft by 42 inches.

   - The length and width of the pool can be any two numbers that have a product of 1000.
   - Since 20
   - The surface area of the water is 500 square feet.

   a. Write the volume in factor form to determine possible dimensions for the rectangular prism.

   b. Solve each equation. Confirm your answers using a graphing calculator.

   c. Confirm the roots using a graphing calculator. Let \(Y1 \) and \( Y2 \) on the right side.

   d. After factoring 3k, the polynomial is a perfect square trinomial.

   e. In the factored trinomial, the middle term is equal to 2\(a\) \& \(b\).

   f. Thus, the solution is \(\frac{3}{8}\).
8-9 Perfect Squares

10. \((z + 5)^2 = 47\)

\[\text{SOLUTION:}\]

\[(z + 5)^2 = 47\]

\[z + 5 = \pm\sqrt{47}\]

\[z = -5 \pm \sqrt{47}\]

The roots are \(-5 - \sqrt{47}\) and \(-5 + \sqrt{47}\) or about \(-11.86\) and \(1.86\).

Confirm the roots using a graphing calculator. Let \(Y_1 = (z + 5)^2\) and \(Y_2 = 47\). Use the intersect option from the CALC menu to find the points of intersection.

Thus, the solutions are \(-5 - \sqrt{47}\) and \(-5 + \sqrt{47}\) or about \(-11.86\) and \(1.86\).

11. CCSS REASONING While painting his bedroom, Nick drops his paintbrush off his ladder from a height of 6 feet. Use the formula \(h = -16t^2 + h_0\) to approximate the number of seconds it takes for the paintbrush to hit the floor.

\[\text{SOLUTION:}\]

Let \(h = 0\) feet and \(h_0 = 6\) feet.

\[h = -16t^2 + h_0\]

\[0 = -16t^2 + 6\]

\[16t^2 = 6\]

\[t^2 = 0.375\]

\[t = \pm\sqrt{0.375}\]

\[t \approx \pm 0.6\]

The roots are \(-0.6\) and \(0.6\). The time the paint brush falls cannot be negative. So, it takes about 0.6 second for the paintbrush to hit the floor.

Determine whether each trinomial is a perfect square trinomial. Write yes or no. If so, factor it.

12. \(4x^2 - 42x + 110\)

\[\text{SOLUTION:}\]

The last term is not a perfect square. So, \(4x^2 - 42x + 110\) is not a perfect square trinomial.
8-9 Perfect Squares

13. $16x^2 - 56x + 49$

**SOLUTION:**
The first term is a perfect square.

$16x^2 = (4x)^2$

The last term is a perfect square.

$49 = (7)^2$

The middle term is equal to $2ab$.

$56x = 2(4x)(7)$

So, $16x^2 - 56x + 49$ is a perfect square trinomial.

$16x^2 - 56x + 49 = (4x)^2 - 2(4x)(7) + 7^2$

$= (4x - 7)^2$

14. $81x^2 - 90x + 25$

**SOLUTION:**
The first term is a perfect square.

$81x^2 = (9x)^2$

The last term is a perfect square.

$25 = (5)^2$

The middle term is equal to $2ab$.

$90x = 2(9x)(5)$

So, $81x^2 - 90x + 25$ is a perfect square trinomial.

$81x^2 - 90x + 25 = (9x)^2 - 2(9x)(5) + 5^2$

$= (9x - 5)^2$

15. $x^2 + 26x + 168$

**SOLUTION:**
The last term is not a square. So, $x^2 + 26x + 168$ is not a perfect square trinomial.
8-9 Perfect Squares

Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.

16. $24d^2 + 39d − 18$

**SOLUTION:**

Factor GCF of 3 from each term.

$24d^2 + 39d − 18 = 3(8d^2 + 13d − 6)$

The resulting polynomial is not a perfect square or a difference of squares. Try to factor using the general factoring pattern.

In the factored trinomial, $a = 8$, $b = 13$ and $c = −6$, so $m + p$ is positive and $mp$ is negative. Therefore, $m$ and $p$ must have different signs. List the factors of $8(−6)$ or $−48$ with a sum of 13.

<table>
<thead>
<tr>
<th>Factors of $−48$</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
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<td>−47</td>
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<td>47</td>
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<td>2, −25</td>
<td>−23</td>
</tr>
<tr>
<td>−2, 25</td>
<td>23</td>
</tr>
<tr>
<td>3, −16</td>
<td>−13</td>
</tr>
<tr>
<td>−3, 16</td>
<td>13</td>
</tr>
<tr>
<td>4, −12</td>
<td>−8</td>
</tr>
<tr>
<td>−4, 12</td>
<td>8</td>
</tr>
<tr>
<td>6, −8</td>
<td>−17</td>
</tr>
<tr>
<td>−6, 8</td>
<td>17</td>
</tr>
</tbody>
</table>

The correct factors are $−3$ and $16$.

$24d^2 + 39d − 18$

$= 3(8d^2 + 13d − 6)$

Factor out GCF of 3 from each term.

$= 3(8d^2 + 16d − 3d − 6)$

$m = 16$ and $p = −3$.

$= 3[(8d^2 + 16d) − (3d + 6)]$

Group terms with common factors.

$= 3[8d(d + 2) − 3(d + 2)]$

Factor GCF from each group.

$= 3(8d − 3)(d + 2)$

$(d + 2)$ is the common factor. 
8-9 Perfect Squares

17. \(8x^2 + 10x - 21\)

\textit{SOLUTION:}

The polynomial is not a perfect square or a difference of squares. Try to factor using the general factoring pattern. In the trinomial, \(a = 8\), \(b = 10\) and \(c = -21\), so \(m + p\) is positive and \(mp\) is negative. Therefore, \(m\) and \(p\) must have different signs. List the factors of \(8(-21)\) or \(-168\) with a sum of 10.

<table>
<thead>
<tr>
<th>Factors of (-60)</th>
<th>Sum</th>
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</thead>
<tbody>
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<td>(-167)</td>
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<tr>
<td>(-1, 168)</td>
<td>167</td>
</tr>
<tr>
<td>2, (-84)</td>
<td>(-82)</td>
</tr>
<tr>
<td>(-2, 84)</td>
<td>82</td>
</tr>
<tr>
<td>3, (-56)</td>
<td>(-53)</td>
</tr>
<tr>
<td>(-3, 56)</td>
<td>53</td>
</tr>
<tr>
<td>4, (-42)</td>
<td>(-38)</td>
</tr>
<tr>
<td>(-4, 42)</td>
<td>38</td>
</tr>
<tr>
<td>6, (-28)</td>
<td>(-22)</td>
</tr>
<tr>
<td>(-6, 28)</td>
<td>22</td>
</tr>
<tr>
<td>8, (-21)</td>
<td>(-13)</td>
</tr>
<tr>
<td>(-8, 21)</td>
<td>13</td>
</tr>
<tr>
<td>(-12, 14)</td>
<td>2</td>
</tr>
<tr>
<td>12, (-14)</td>
<td>(-2)</td>
</tr>
</tbody>
</table>

There are no factors of \(8(-21)\) or \(-168\) with a sum of 10.

So, this trinomial is prime.
8-9 Perfect Squares

18. \(2b^2 + 12b - 24\)

**SOLUTION:**
The greatest common factor of each term is 2.

\[2b^2 + 12b - 24 = 2(b^2 + 6b - 12)\]

The resulting polynomial is not a perfect square or a difference of squares. Try to factor using the general factoring pattern.

In the trinomial, \(b = 6\) and \(c = -12\), so \(m + p\) is positive and \(mp\) is negative. Therefore, \(m\) and \(p\) must have different signs. List the factors of \(-12\) with a sum of 6.

<table>
<thead>
<tr>
<th>Factors of (-12)</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, (-12)</td>
<td>(-11)</td>
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<tr>
<td>(-1, 12)</td>
<td>11</td>
</tr>
<tr>
<td>2, (-6)</td>
<td>(-4)</td>
</tr>
<tr>
<td>(-2, 6)</td>
<td>4</td>
</tr>
<tr>
<td>3, (-4)</td>
<td>(-1)</td>
</tr>
<tr>
<td>(-3, 4)</td>
<td>1</td>
</tr>
</tbody>
</table>

There are no factors of \(-12\) with a sum of 6. Thus \(b^2 + 6b - 12\) is prime.

Therefore, \(2(b^2 + 6b - 12)\) is factored form form.

19. \(8y^2 - 200z^2\)

**SOLUTION:**
Factor out the common factor 8.

\[8y^2 - 200z^2 = 8(y^2 - 25z^2)\]

Then \(y^2 - 25z^2\) is a difference of squares.

\[8y^2 - 200z^2 = 8(y^2 - 25z^2) = 8(y - 5z)(y + 5z)\]

20. \(16a^2 - 121b^2\)

**SOLUTION:**
The polynomial is difference of squares.

\[16a^2 - 121b^2 = (4a)^2 - (11b)^2 = (4a - 11b)(4a + 11b)\]
8-9 Perfect Squares

21. \(12m^3 - 22m^2 - 70m\)

**SOLUTION:**

\[
12m^3 - 22m^2 - 70m = 2m\left(6m^2 - 11m - 35\right)
\]

The polynomial is not a perfect square or a difference of squares. Try to factor using the general factoring pattern. In the factored trinomial, \(a = 6, b = -11\) and \(c = -35\), so \(m + p\) is negative and \(mp\) is negative. Therefore, \(m\) and \(p\) must have different signs. List the factors of \(6(-35)\) or \(-210\) with a sum of \(-11\).

<table>
<thead>
<tr>
<th>Factors of (-210)</th>
<th>Sum</th>
</tr>
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<tbody>
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<td>-209</td>
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<tr>
<td>-1, 210</td>
<td>209</td>
</tr>
<tr>
<td>2, -105</td>
<td>-103</td>
</tr>
<tr>
<td>-2, 105</td>
<td>103</td>
</tr>
<tr>
<td>3, -70</td>
<td>-67</td>
</tr>
<tr>
<td>-3, 70</td>
<td>67</td>
</tr>
<tr>
<td>5, -42</td>
<td>-37</td>
</tr>
<tr>
<td>-5, 42</td>
<td>37</td>
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<tr>
<td>6, -35</td>
<td>-29</td>
</tr>
<tr>
<td>-6, 35</td>
<td>29</td>
</tr>
<tr>
<td>7, -30</td>
<td>-23</td>
</tr>
<tr>
<td>-7, 30</td>
<td>23</td>
</tr>
<tr>
<td>-10, 21</td>
<td>11</td>
</tr>
<tr>
<td>10, -21</td>
<td>-11</td>
</tr>
</tbody>
</table>

The correct factors are 10 and \(-21\).

\[
12m^3 - 22m^2 - 70m
= 2m\left(6m^2 - 11m - 35\right) \\
= 2m\left(6m^2 - 2\cdot11m + 10m - 35\right) \\
= 2m\left[\left(6m^2 - 2\cdot11m\right) + (10m - 35)\right] \\
= 2m\left[3m(2m-7) + 5(2m-7)\right] \\
= 2m(2m-7)(3m+5)
\]

Factor GCF of \(2m\).

\(m = -21\) and \(p = 10\).

Group terms with common factors.

Factor GCF from each group.

\((2m-7)\) is the common factor.
22. 8c^2 - 88c + 242

**SOLUTION:**
After factoring out 2, the polynomial is a perfect square.

\[
8c^2 - 88c + 242 = 2(4c^2 - 44c + 121) \quad \text{Factor}
\]

\[
= 2[(2c)^2 - 2(2c)(11) + 11^2] \quad \text{Perfect square trinomial}
\]

\[
= 2(2c - 11)^2 \quad \text{Factor.}
\]

23. 12x^2 - 84x + 147

**SOLUTION:**
After factoring out a 3, the polynomial is a perfect square.

\[
12x^2 - 84x + 147 = 3(4x^2 - 28x + 49) \quad \text{Factor the GCF}
\]

\[
= 3[(2x)^2 - 2(2x)(7) + 7^2] \quad \text{perfect square trinomial}
\]

\[
= 3(2x - 7)^2 \quad \text{Factor.}
\]

24. w^4 - w^2

**SOLUTION:**
The polynomial is a difference of squares.

\[
w^4 - w^2 = w^2(w^2 - 1) \quad w^2 \text{is the GCF of the terms.}
\]

\[
= w^2(w^2 - 1^2) \quad \text{Write in the form } a^2 - b^2.
\]

\[
= w^2(w - 1)(w + 1) \quad \text{Factor the difference of squares.}
\]

25. 12p^3 - 3p

**SOLUTION:**
After factoring out 3p, the polynomial is a difference of squares.

\[
12p^3 - 3p = 3p(4p^2 - 1) \quad 3p \text{ is GCF for the terms.}
\]

\[
= 3p[(2p)^2 - 1^2] \quad \text{Write in the form } a^2 - b^2.
\]

\[
= 3p(2p + 1)(2p - 1) \quad \text{Factor the difference of squares.}
\]
8-9 Perfect Squares

26. \(16q^3 - 48q^2 + 36q\)

**SOLUTION:**
After factoring out \(4q\), the polynomial is a perfect square.

\[16q^3 - 48q^2 + 36q = 4q\left(4q^2 - 12q + 9\right)\]

Factor out the GCF.

\[= 4q\left[(2q)^2 - 2(2q)(3) + 3^2\right]\]

perfect square trinomial.

\[= 4q(2q - 3)^2\]

Factor.
27. $4t^3 + 10t^2 - 84t$

**SOLUTION:**
Factor out the GCF.

$$4t^3 + 10t^2 - 84t = 2t\left(2t^2 + 5t - 42\right)$$

The polynomial is not a perfect square or a difference of squares. Try to factor using the general factoring pattern.

In this trinomial, $a = 2$, $b = 5$ and $c = -42$, so $m + p$ is positive and $mp$ is negative. List the positive and negative factors of $2(-42)$ or $-84$, and identify the factors which sum to 5.

<table>
<thead>
<tr>
<th>Factors of $-84$</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
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<tr>
<td>$-1, 84$</td>
<td>83</td>
</tr>
<tr>
<td>2, $-42$</td>
<td>$-40$</td>
</tr>
<tr>
<td>$-2, 42$</td>
<td>40</td>
</tr>
<tr>
<td>3, $-28$</td>
<td>$-25$</td>
</tr>
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<td>$-3, 28$</td>
<td>25</td>
</tr>
<tr>
<td>4, $-21$</td>
<td>$-17$</td>
</tr>
<tr>
<td>$-4, 21$</td>
<td>17</td>
</tr>
<tr>
<td>6, $-14$</td>
<td>$-8$</td>
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<td>7, $-12$</td>
<td>$-5$</td>
</tr>
<tr>
<td>$-7, 12$</td>
<td>5</td>
</tr>
</tbody>
</table>

The correct factors are 12 and $-7$.

$$2t\left(2t^2 + 5t - 42\right)$$

$$= 2t\left(2t^2 + 12t - 7t - 42\right) \quad m = 12 \text{ and } p = -7$$

$$= 2t\left[\left(2t^2 + 12t\right) - (7t + 42)\right] \quad \text{Group terms with common factors.}$$

$$= 2t\left[2t(t + 6) - 7(t + 6)\right] \quad \text{Factor the GCF from each group.}$$

$$= 2t(t + 6)(2t - 7) \quad (t + 6) \text{ is the common factor.}$$
28. \( x^3 + 2x^2y - 4x - 8y \)

**SOLUTION:**
There are four terms, so factor by grouping.

\[
x^3 + 2x^2y - 4x - 8y = (x^3 + 2x^2y) - (4x + 8y) \text{ Group terms.}
= x^2(x + 2y) - 4(x + 2y) \text{ Factor GCF.}
= (x + 2y)(x^2 - 4) \text{ (x + 2y) is the common factor}
= (x + 2y)(x - 2)(x + 2) \text{ Factor } x^2 - 4.
\]

29. \( 2a^2b^2 - 2a^2 - 2ab^3 + 2ab \)

**SOLUTION:**
There are four terms, so factor by grouping.

\[
2a^2b^2 - 2a^2 - 2ab^3 + 2ab = 2a(ab^2 - a - b^3 + b) \text{ 2a is the GCF of the terms.}
= 2a[(ab^2 - a) - (b^3 - b)] \text{ Group terms with common factors.}
= 2a[a(b^2 - 1) - b(b^2 - 1)] \text{ Factor GCF from each group.}
= 2a(a - b)(b^2 - 1) \text{ (b^2 - 1) is the common factor.}
= 2a(a - b)(b + 1)(b - 1) \text{ Factor } b^2 - 1 \text{ adifference of perfect square.}
\]

30. \( 2r^3 - r^2 - 72r + 36 \)

**SOLUTION:**
There are four terms, so factor by grouping.

\[
2r^3 - r^2 - 72r + 36 = (2r^3 - r^2) - (72r - 36) \text{ Group terms with common factors.}
= r^2(2r - 1) - 36(2r - 1) \text{ Factor GCF from each group.}
= (r^2 - 36)(2r - 1) \text{ (2r - 1) is the common factor.}
= (r - 6)(r + 6)(2r - 1) \text{ Factor } r^2 - 36.
\]
8-9 Perfect Squares

31. $3k^3 - 24k^2 + 48k$

**SOLUTION:**
After factoring $3k$, the polynomial is a perfect square trinomial.

$$3k^3 - 24k^2 + 48k = 3k(k^2 - 8k + 16)$$

$$= 3k(k^2 - 2(k)(4) + 4^2)$$  
perfect square trinomial.

$$= 3k(k - 4)^2$$  
Factor.

32. $4c^4d - 10c^3d + 4c^2d^3 - 10cd^3$

**SOLUTION:**
There are four terms, so factor by grouping.

$$4c^4d - 10c^3d + 4c^2d^3 - 10cd^3$$

$$= 2cd(2c^3 - 5c^2 + 2cd^2 - 5d^2)$$  
$2cd$ is the GCF of the terms.

$$= 2cd[(2c^3 - 5c^2) + (2cd^2 - 5d^2)]$$  
Group terms.

$$= 2cd[c^2(2c - 5) + d^2(2c - 5)]$$  
Factor GCF.

$$= 2cd[(c^2 + d^2)(2c - 5)]$$  
$(2c - 5)$ is the common factor

$$= 2cd(c^2 + d^2)(2c - 5)$$  
Simplify.

33. $g^2 + 2g - 3h^2 + 4h$

**SOLUTION:**
The GCF of the terms $g^2$, $2g$, $-3h^2$, and $4h$ is 1, so there is no GCF to factor out.

Since there are four terms, consider factor by grouping. Only the pairs of the first two terms and the last two terms have GCFs other than 1, so try factoring using this grouping.

$$g^2 + 2g - 3h^2 + 4h = (g^2 + 2g) + (-3h^2 + 4h)$$

$$= g(g + 2) + h(-3h + 4)$$

There is no common binomial factor, so this polynomial cannot be written as a product.
Thus the polynomial $g^2 + 2g - 3h^2 + 4h$ cannot be factored.

It is prime.
Solve each equation. Confirm your answers using a graphing calculator.

34. \(4m^2 - 24m + 36 = 0\)

**SOLUTION:**
The GCF of the terms is 4, so factor it out.

\[
4m^2 - 24m + 36 = 0 \quad \text{Original equation}
\]
\[
4\left(m^2 - 6m + 9\right) = 0 \quad \text{4 is the GCF.}
\]
\[
4\left(m^2 - 2(m)(3) + 3^2\right) = 0 \quad \text{perfect square trinomial}
\]
\[
4(m - 3)^2 = 0 \quad \text{Factor.}
\]

\[m - 3 = 0\]
\[m = 3\]

The root is 3.

Confirm the root using a graphing calculator. Let \(Y_1 = 4m^2 - 24m + 36\) and \(Y_2 = 0\). Use the **intersect** option from the **CALC** menu to find the points of intersection.

![Graph showing intersection at x=3]

Thus, the solution is 3.
8-9 Perfect Squares

35. \((y - 4)^2 = 7\)

**SOLUTION:**

\[(y - 4)^2 = 7\] \hspace{1cm} \text{Original equation}

\[y - 4 = \pm \sqrt{7}\] \hspace{1cm} \text{Square Root Property}

\[y = 4 \pm \sqrt{7}\] \hspace{1cm} \text{Add 4 to each side.}

The roots are \(4 - \sqrt{7}\) and \(4 + \sqrt{7}\) or about 1.35 and 6.65.

Confirm the roots using a graphing calculator. Let Y1 = \((y - 4)^2\) and Y2 = 7. Use the **intersect** option from the **CALC** menu to find the points of intersection.

Thus the solutions are \(4 - \sqrt{7}\) and \(4 + \sqrt{7}\).
36. \( a^2 + \frac{10}{7}a + \frac{25}{49} = 0 \)

**Solution:**
The GCF of the terms is 1. The polynomial \( a^2 + \frac{10}{7}a + \frac{25}{49} \) is a perfect square trinomial.

\[
\begin{align*}
\text{Original equation} & \quad a^2 + \frac{10}{7}a + \frac{25}{49} = 0 \\
\text{Perfect square trinomial} & \quad a^2 + 2(a)\left(\frac{5}{7}\right) + \left(\frac{5}{7}\right)^2 = 0 \\
\text{Factor} & \quad \left(a + \frac{5}{7}\right)^2 = 0 \\
\text{Root} & \quad a = -\frac{5}{7}
\end{align*}
\]

The root is \(-\frac{5}{7}\) or about \(-0.71\).

Confirm the roots using a graphing calculator. Let \(Y_1 = a^2 + \frac{10}{7}a + \frac{25}{49}\) and \(Y_2 = 0\). Use the intersect option from the CALC menu to find the points of intersection.

Thus, the solution is \(-\frac{5}{7}\).
8-9 Perfect Squares

37. \( x^2 - \frac{3}{2}x + \frac{9}{16} = 0 \)

**SOLUTION:**

The GCF of the terms is 1. The polynomial \( x^2 - \frac{3}{2}x + \frac{9}{16} \) is a perfect square trinomial.

\[
\begin{align*}
x^2 - \frac{3}{2}x + \frac{9}{16} &= 0 & \text{Original equation} \\
x^2 - 2\left(\frac{3}{4}\right)x + \left(\frac{3}{4}\right)^2 &= 0 & \text{perfect square trinomial} \\
\left(x - \frac{3}{4}\right)^2 &= 0 & \text{Factor.}
\end{align*}
\]

\[ x - \frac{3}{4} = 0 \]
\[ x = \frac{3}{4} \]

The root is \( \frac{3}{4} \) or 0.75.

Confirm the roots using a graphing calculator. Let \( Y_1 = x^2 - \frac{3}{2}x + \frac{9}{16} \) and \( Y_2 = 0 \). Use the **intersect** option from the **CALC** menu to find the points of intersection.

Thus, the solution is \( \frac{3}{4} \) or 0.75.
38. \( x^2 + 8x + 16 = 25 \)

**SOLUTION:**

The GCF of the terms is 1. The polynomial \( x^2 + 8x + 16 \) is a perfect square trinomial.

\[
x^2 + 8x + 16 = 25 \quad \text{Original equation}
\]

\[
x^2 + 2(x)(4) + 4^2 = 25 \quad \text{perfect square trinomial}
\]

\[
(x + 4)^2 = 25 \quad \text{Factor.}
\]

\[
x + 4 = \pm \sqrt{25} \quad \text{Square Root Property}
\]

\[
x = -4 \pm 5 \quad \text{Subtract 4 from each side.}
\]

\[
x = -4 - 5 \quad \text{or} \quad x = -4 + 5
\]

\[
x = -9 \quad \text{or} \quad x = 1
\]

The roots are −9 and 1.

Confirm the roots using a graphing calculator. Let \( Y_1 = x^2 + 8x + 16 \) and \( Y_2 = 25 \). Use the *intersect* option from the \text{CALC} menu to find the points of intersection.

Thus, the solutions are −9 and 1.


8-9 Perfect Squares

39. $5x^2 - 60x = -180$

**SOLUTION:**
Write with 0 on the right side and factor out GCF of 5.

\[
5x^2 - 60x = -180 \quad \text{Original equation}
\]

\[
5x^2 - 60x + 180 = 0 \quad \text{Add 180 to each side.}
\]

\[
5(x^2 - 12x + 36) = 0 \quad \text{5 is the GCF of the terms}
\]

\[
5(x^2 - 2(x)(6) + 6^2) = 0 \quad \text{perfect square trinomial}
\]

\[
5(x - 6)^2 = 0 \quad \text{Factor}
\]

\[
x - 6 = 0
\]

\[
x = 6
\]

The root is 6.

Confirm the root using a graphing calculator. Let $Y_1 = 5x^2 - 60x$ and $Y_2 = -180$. Use the **intersect** option from the **CALC** menu to find the points of intersection.

Thus, the solution is 6.
8-9 Perfect Squares

40. \(4x^2 = 80x - 400\)

**SOLUTION:**
Rewrite the trinomial with 0 on the right side. The GCF of the terms is 4, so factor it out.

\[
4x^2 = 80x - 400 \quad \text{Original equation}
\]

\[
4x^2 - 80x + 400 = 0 \quad \text{Add 400 to each side.}
\]

\[
4(x^2 - 20x + 100) = 0 \quad 4 \text{ is the GCF of the terms}
\]

\[
4(x^2 - 2(x)(10) + 10^2) = 0 \quad \text{perfect square trinomial}
\]

\[
4(x - 10)^2 = 0 \quad \text{Factor.}
\]

\[
x - 10 = 0
\]

\[
x = 10
\]

The root is 10.

Confirm the roots using a graphing calculator. Let \(Y_1 = 4x^2\) and \(Y_2 = 80x - 400\). Use the `intersect` option from the `CALC` menu to find the points of intersection.

Thus, the solution is 10.
8-9 Perfect Squares

41. $9 - 54x = -81x^2$

SOLUTION:
Rewrite the trinomial with 0 on the right side. The GCF of the terms is 9, so factor it out.

\[
\begin{align*}
9 - 54x &= -81x^2 & \text{Original equation} \\
81x^2 - 54x + 9 &= 0 & \text{Add } 81x^2 \text{ to each side.} \\
9(9x^2 - 6x + 1) &= 0 & \text{9 is the GCF of the terms} \\
9[(3x)^2 - 2(3x)(1) + 1]^2 &= 0 & \text{perfect square trinomial} \\
9(3x - 1)^2 &= 0 & \text{Factor.}
\end{align*}
\]

\[
\begin{align*}
3x - 1 &= 0 \\
3x &= 1 \\
x &= \frac{1}{3}
\end{align*}
\]

The root is $\frac{1}{3}$ or about 0.33.

Confirm the roots using a graphing calculator. Let $Y_1 = 9 - 54x$ and $Y_2 = -81x^2$. Use the intersect option from the CALC menu to find the points of intersection.

Thus, the solution is $\frac{1}{3}$. 
8-9 Perfect Squares

42. \(4c^2 + 4c + 1 = 15\)

\textbf{SOLUTION:}

The GCF of the terms is 1. The polynomial \(4c^2 + 4c + 1\) is a perfect square trinomial.

\[
4c^2 + 4c + 1 = 15 \quad \text{Original equation}
\]
\[
(2c)^2 + 2(2c)(1) + 1^2 = 15 \quad \text{Perfect square trinomial}
\]
\[
(2c + 1)^2 = 15 \quad \text{Factor.}
\]
\[
2c + 1 = \pm \sqrt{15} \quad \text{Square Root Property}
\]
\[
2c = -1 \pm \sqrt{15} \quad \text{Subtract 1 from each side.}
\]
\[
c = \frac{-1 \pm \sqrt{15}}{2} \quad \text{Divide each side by 2.}
\]

The roots are \(\frac{-1 - \sqrt{15}}{2}\) and \(\frac{-1 + \sqrt{15}}{2}\) or about \(-2.44\) and \(1.44\).

Confirm the roots using a graphing calculator. Let \(Y_1 = 4c^2 + 4c + 1\) and \(Y_2 = 15\). Use the \textbf{intersect} option from the \textbf{CALC} menu to find the points of intersection.

\[
\text{Intersection} \quad X = 2.4364917 \quad Y = 15
\]
\[
[-5, 5] \text{ scl: 1 by } [-10, 20] \text{ scl: 3}
\]

\[
\text{Intersection} \quad X = 1.4364917 \quad Y = 15
\]
\[
[-5, 5] \text{ scl: 1 by } [-10, 20] \text{ scl: 3}
\]

Thus, the solutions are \(\frac{-1 - \sqrt{15}}{2}\) and \(\frac{-1 + \sqrt{15}}{2}\).
**8-9 Perfect Squares**

43. \( x^2 - 16x + 64 = 6 \)

**SOLUTION:**

The GCF of the terms is 1. The polynomial \( x^2 - 16x + 64 \) is a perfect square trinomial.

\[
\begin{align*}
\quad & x^2 - 16x + 64 = 6 & \text{Original equation} \\
\quad & x^2 - 2(x)(8) + 8^2 = 6 & \text{Perfect square trinomial} \\
\quad & (x - 8)^2 = 6 & \text{Factor.} \\
\quad & x - 8 = \pm \sqrt{6} & \text{Square Root Property} \\
\quad & x = 8 \pm \sqrt{6} & \text{Add 8 to each side.}
\end{align*}
\]

The roots are \( 8 - \sqrt{6} \) and \( 8 + \sqrt{6} \) or 5.55 and 10.45.

Confirm the roots using a graphing calculator. Let \( Y_1 = x^2 - 16x + 64 \) and \( Y_2 = 6 \). Use the **intersect** option from the **CALC** menu to find the points of intersection.

Thus, the solutions are \( 8 - \sqrt{6} \) and \( 8 + \sqrt{6} \).
44. PHYSICAL SCIENCE For an experiment in physics class, a water balloon is dropped from the window of the school building. The window is 40 feet high. How long does it take until the balloon hits the ground? Round to the nearest hundredth.

SOLUTION:
Use the formula $h = -16t^2 + h_0$ to approximate the number of seconds it takes for the balloon to hit the ground. At ground level, $h = 0$ and the initial height is 40, so $h_0 = 40$.

\[
\begin{align*}
  h &= -16t^2 + h_0 & \text{Original Formula} \\
  0 &= -16t^2 + 40 & \text{Replace } h \text{ with 0 and } h_0 \text{ with 40.} \\
  16t^2 &= 40 & \text{Add } 16t^2 \text{ to each side.} \\
  t^2 &= 2.5 & \text{Divide each side by 16.} \\
  t &= \pm \sqrt{2.5} & \text{Use the Square Root Property.} \\
  t &\approx \pm 1.58 & \text{Use a calculator.}
\end{align*}
\]

The roots are $-1.58$ and $1.58$. The time the balloon falls cannot be negative.

So, it takes about 1.58 seconds for the balloon to hit the ground.

45. SCREENS The area $A$ in square feet of a projected picture on a movie screen can be modeled by the equation $A = 0.25d^2$, where $d$ represents the distance from a projector to a movie screen. At what distance will the projected picture have an area of 100 square feet?

SOLUTION:
Use the formula to find $d$ when $A = 100$.

\[
\begin{align*}
  A &= 0.25d^2 & \text{Original formula} \\
  100 &= 0.25d^2 & \text{Replace } A \text{ with 100.} \\
  400 &= d^2 & \text{Divide each side by 0.25.} \\
  \pm 20 &= d & \text{Use the Square Root Property}
\end{align*}
\]

The roots are $-20$ and $20$. The distance cannot be negative.

So, to have an area of 100 square feet, the projected picture will be 20 feet from the projector.
46. GEOMETRY The area of a square is represented by $9x^2 - 42x + 49$. Find the length of each side.

**SOLUTION:**

\[
A = s^2 \\
9x^2 - 42x + 49 = s^2 \\
(3x)^2 - 2(3x)(7) + 7^2 = s^2 \\
(3x - 7)^2 = s^2 \\
|3x - 7| = s
\]

The length of each side of the square is $|3x - 7|$.

47. GEOMETRY The area of a square is represented by $16x^2 + 40x + 25$. Find the length of each side.

**SOLUTION:**

\[
A = s^2 \\
16x^2 + 40x + 25 = s^2 \\
(4x)^2 + 2(4x)(5) + 5^2 = s^2 \\
(4x + 5)^2 = s^2 \\
|4x + 5| = s
\]

The length of each side of the square is $|4x + 5|$.

48. GEOMETRY The volume of a rectangular prism is represented by the expression $8y^3 + 40y^2 + 50y$. Find the possible dimensions of the prism if the dimensions are represented by polynomials with integer coefficients.

**SOLUTION:**

Write the volume in factor form to determine possible dimensions for the rectangular prism.
The GCF for the terms is $2y$, so factor it out first.

\[
V = 8y^3 + 40y^2 + 50y \\
= 2y(4y^2 + 20y + 25) \\
= 2y[(2y)^2 + 2(2y)(5) + 5^2] \\
= 2y(2y + 5)(2y + 5)
\]

So, the possible dimensions are $2y$, $2y + 5$, and $2y + 5$.

49. POOLS Ichiro wants to buy an above-ground swimming pool for his yard. Model A is 42 inches deep and holds 1750 cubic feet of water. The length of the rectangular pool is 5 feet more than the width.

a. What is the surface area of the water?

b. What are the dimensions of the pool?

c. Model B pool holds twice as much water as Model A. What are some possible dimensions for this pool?
8-9 Perfect Squares

d. Model C has length and width that are both twice as long as Model A, but the height is the same. What is the ratio of the volume of Model A to Model C?

**SOLUTION:**

a. Let $w = \text{the width of the pool and } w + 5 = \text{the length of the pool.}$ There are 12 inches in a foot. So the pool is $42 \div 12$ or 3.5 feet deep.

$$V = \ell w h$$
$$1750 = (w + 5)(w)(3.5)$$
$$1750 = (w^2 + 5w)(3.5)$$
$$1750 = 3.5w^2 + 17.5w$$

0 = 3.5w^2 + 3.5w - 1750

0 = 3.5(w^2 + 5w - 500)

0 = 3(w^2 + 25w - 20w - 500)

0 = 3(w - 20)(w + 25)

$$w - 20 = 0 \quad \text{or} \quad w + 25 = 0$$

$$w = 20 \quad \text{or} \quad w = -25$$

The roots are -25 and 20. The width cannot be negative, so the width is 20 feet and the length is 25 feet.

$$A = \ell w$$

$$= 25(20)$$

$$= 500$$

The surface area of the water is 500 square feet.

b. The pool is 20 feet wide by 25 feet long by 42 inches deep.

c. Sample answer: Model B pool holds $2 \times 1750$ or 3500 cubic feet of water.

$$V = \ell w h$$

$$3500 = \ell w (3.5)$$

$$1000 = \ell w$$

The length and width of the pool can be any two numbers that have a product of 1000. Since $20 \times 50 = 1000$, the dimensions of Model B pool could be 20 ft by 50 ft by 42 inches.

d. The width of Model C is $2 \times 20$ or 40 feet. The length of Model C is $2 \times 25$ or 50 feet. The depth is still 42 inches, or 3.5 feet.

$$V = \ell w h$$

$$= (50)(40)(3.5)$$

$$= 7000$$
8-9 Perfect Squares

The ratio of the volume of Model A to Model C is \( \frac{1750}{7000} = \frac{1}{4} \).

50. GEOMETRY Use the rectangular prism.

![Rectangular Prism Diagram]

\( t = 14 \)

\( 8 \)

\( 4 \)

a. Write an expression for the height and width of the prism in terms of the length, \( \ell \).

b. Write a polynomial for the volume of the prism in terms of the length.

SOLUTION:

a. The height is 8 or the length minus 6. Let \( \ell = \) the length. Then, the height is \( \ell - 6 \). The width is 4 or the length minus 10. Then, the width is \( \ell - 10 \).

b. 

\[
V = lwh \\
= \ell (\ell - 10)(\ell - 6) \\
= \ell (\ell^2 - 6\ell - 10\ell + 60) \\
= \ell (\ell^2 - 16\ell + 60) \\
= \ell^3 - 16\ell^2 + 60\ell
\]
51. CCSS PRECISION A zoo has an aquarium shaped like a rectangular prism. It has a volume of 180 cubic feet. The height of the aquarium is 9 feet taller than the width, and the length is 4 feet shorter than the width. What are the dimensions of the aquarium?

**SOLUTION:**

a. Write a polynomial to represent the volume.

The dimensions are \( \ell = w - 4 \), \( h = w + 9 \), and \( w \).

\[
V = \ell wh \\
= (w-4)(w+9)(w) \\
= (w^2 + 5w - 36)(w) \\
= w^3 + 5w^2 - 36w
\]

The volume is 180 cubic feet. Substitute 180 for \( V \) and get 0 on one side of the equation.

\[
V = w^3 + 5w^2 - 36w \\
180 = w^3 + 5w^2 - 36w \\
0 = w^3 + 5w^2 - 36w - 180
\]

Use factor by grouping to factor the right side of the equation.

\[
0 = (w^3 + 5w^2) - (36w + 180) \quad \text{Group terms.} \\
0 = w^2(w + 5) - 36(w + 5) \quad \text{Factor GCF.} \\
0 = (w^2 - 36)(w + 5) \quad (w + 5) \text{ is the common factor.} \\
0 = (w + 6)(w - 6)(w + 5) \quad \text{Difference of perfect square.}
\]

The value of \( w \) can be \(-6\), \(6\), or \(-5\). However, no measurement of length can be negative, so \( w = 6 \), \( \ell = w - 4 \) or 2, and \( h = w + 9 \) or 15.

Therefore, the aquarium is 6 feet wide, 2 feet long, and 15 feet high.
52. **ELECTION** For the student council elections, Franco is building the voting box shown with a volume of 96 cubic inches. What are the dimensions of the voting box?

![Voting Box Diagram]

**SOLUTION:**
Write a polynomial to represent the volume.

The dimensions are $\ell = h + 8$, $w = h - 2$, and $h$.

\[
V = \ell wh \\
= (h + 8)(h - 2)(h) \\
= (h^2 + 6h - 16)(h) \\
= h^3 + 6h^2 - 16h
\]

The volume is 96 cubic inches. Substitute 96 for $V$ and get 0 on one side of the equation.

\[
V = h^3 + 6h^2 - 16h \\
96 = h^3 + 6h^2 - 16h \\
0 = h^3 + 6h^2 - 16h - 96
\]

Use factor by grouping to factor the right side of the equation.

\[
0 = h^3 + 6h^2 - 16h - 96 \\
0 = h^2(h + 6) - 16(h + 6) \\
0 = (h^2 - 16)(h + 6) \\
0 = (h + 4)(h - 4)(h + 6)
\]

The value of $h$ can be $-4$, $4$, or $-6$. However, no measurement of length can be negative, so $h = 4$, $\ell = h + 8$ or 12, and $w = h - 2$ or 2.

So, the voting box is 4 inches high, 12 inches long and 2 inches wide.
53. **ERROR ANALYSIS** Debbie and Adriano are factoring the expression $x^8 - x^4$ completely. Is either of them correct? Explain your reasoning.

**SOLUTION:**

$$x^8 - x^4 = x^4(x^4 - 1)$$

$$= x^4(x^2 + 1)(x^2 - 1)$$

$$= x^4(x^2 + 1)(x - 1)(x + 1)$$

Adriano is correct. Debbie did not factor the expression completely. She should have factored $x^2 - 1$ into $(x - 1)(x + 1)$.

54. **CHALLENGE** Factor $x^{n+6} + x^{n+2} + x^n$ completely.

**SOLUTION:**

The greatest common factor of each term is $x^n$.

$$x^{n+6} + x^{n+2} + x^n = x^n(x^6 + x^2 + 1)$$
8-9 Perfect Squares

55. OPEN ENDED Write a perfect square trinomial equation in which the coefficient of the middle term is negative and the last term is a fraction. Solve the equation.

SOLUTION:
Find $a$, $b$, and $c$ in the trinomial $ax^2 + bx + c$, so that the coefficient of the middle term is negative and the last term is a fraction.
Choose a fraction for the last term where the numerator and denominator are perfect squares. Let $a = 1$. The coefficient of the middle terms is $-2\sqrt{\frac{9}{4}}$, or $-3$. The trinomial will then be $x^2 - 3x + \frac{9}{4}$.

$x^2 - 3x + \frac{9}{4} = 0$

$x^2 - 3x + \frac{9}{4} = 0$

$x^2 - 2\left(x\right)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 = 0$

$x - \frac{3}{2} = 0$

$x = \frac{3}{2}$

The solution is $\frac{3}{2}$.

56. REASONING Find a counterexample to the following statement.

A polynomial equation of degree three always has three real solutions.

SOLUTION:
Sample answer: You can create an equation of degree 3 that has only one solution by choosing a linear factor and a quadratic factor that cannot equal 0. For example, the factor $(x^2 + 1)$ cannot equal zero for any real value of $x$. The product of the linear factor $(x + 1)$ and $(x^2 + 1)$ is $x^3 + x^2 + x + 1$. The polynomial equation $x^3 + x^2 + x + 1 = 0$ only has one solution since only the factor $(x + 1)$ can equal zero when $x = -1$. Thus, this polynomial equation of degree 3 has only one solution. [Since the only solution for $x^3 = 1$ is $x = 1$, another counterexample could be $x^3 - 1 = 0$.]

57. CCSS REGULARITY Explain how to factor a polynomial completely.

SOLUTION:
First look for a GCF in all the terms and factor the GCF out of all the terms. Then, if the polynomial has two terms, check if the terms are the differences of squares and factor if so. If the polynomial has three terms, check if the trinomial will factor into two binomial factors or if it is a perfect square trinomial and factor if so. If the polynomial has four or more terms, factor by grouping. If the polynomial does not have a GCF and cannot be factored, the polynomial is a prime polynomial.
8-9 Perfect Squares

58. WHICH ONE DOESN’T BELONG? Identify the trinomial that does not belong. Explain.

\[
\begin{align*}
4x^2 - 36x + 81 & \\
25x^2 + 10x + 1 & \\
4x^2 + 10x + 4 & \\
9x^2 - 24x + 16 & \\
\end{align*}
\]

**SOLUTION:**
Identify the first, last and middle terms. Write them as perfect squares if possible.

<table>
<thead>
<tr>
<th>Trinomial</th>
<th>First Term</th>
<th>Last Term</th>
<th>Middle Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4x^2 - 36x + 81)</td>
<td>(4x^2 = (2x)^2)</td>
<td>(81 = 9^2)</td>
<td>(-36x = -2 \cdot 2x \cdot 9)</td>
</tr>
<tr>
<td>(25x^2 + 10x + 1)</td>
<td>(25x^2 = (5x)^2)</td>
<td>(1 = 1^2)</td>
<td>(10x = 2 \cdot 5x \cdot 1)</td>
</tr>
<tr>
<td>(4x^2 + 10x + 4)</td>
<td>(4x^2 = (2x)^2)</td>
<td>(4 = 2^2)</td>
<td>(10x = 2 \cdot 4x \cdot 2 = 16x)</td>
</tr>
<tr>
<td>(9x^2 - 24x + 16)</td>
<td>(9x^2 = (3x)^2)</td>
<td>(16 = 4^2)</td>
<td>(-24x = -2 \cdot 3x \cdot 4)</td>
</tr>
</tbody>
</table>

In each of the polynomials, \(4x^2 - 36x + 81\), \(25x^2 + 10x + 1\), and \(9x^2 - 24x + 16\), the first and last terms are perfect squares and the middle terms are \(2ab\). So, they are perfect square trinomials. The trinomial \(4x^2 + 10x + 4\) is not a perfect square because the middle term is not \(2ab\).

59. OPEN ENDED Write a binomial that can be factored using the difference of two squares twice. Set your binomial equal to zero and solve the equation.

**SOLUTION:**
Find \(a\) and \(b\) so that the binomial \(a^2 - b^2\) that can be factored using the difference of two squares twice. Let \(b = 1\). Then choose a such that it is raised to the 4th power. Let \(a = x^2\). Then \(a^2 - b^2 = x^4 - 1\).

\[
x^4 - 1 = (x^2 + 1)(x^2 - 1) \\
= (x^2 + 1)(x + 1)(x - 1)
\]

\( (x^2 + 1)(x + 1)(x - 1) = 0 \)

\( x + 1 = 0 \) \text{ or } \( x - 1 = 0 \)

\( x = -1 \) \text{ or } \( x = 1 \)

The solutions are \(-1\) and \(1\).

60. WRITING IN MATH Explain how to determine whether a trinomial is a perfect square trinomial.

**SOLUTION:**
Determine if the first and last terms are perfect squares. Then determine if the middle term is equal to \(\pm 2\) times the product of the principal square roots of the first and last terms. If these three criteria are met, the trinomial is a perfect square trinomial.
8-9 Perfect Squares

61. What is the solution set for the equation \((x - 3)^2 = 25\)?

A \{-8, 2\}
B \{-2, 8\}
C \{4, 14\}
D \{-4, 14\}

**SOLUTION:**
\[(x - 3)^2 = 25\]
\[x - 3 = \pm\sqrt{25}\]
\[x = 3 \pm 5\]
\[x = 3 - 5 \quad \text{or} \quad x = 3 + 5\]
\[x = -2 \quad \text{or} \quad x = 8\]

The solutions are \(-2\) and \(8\), so the correct choice is B.

62. **SHORT RESPONSE** Write an equation in slope-intercept form for the graph shown.

![Graph](image)

**SOLUTION:**
The line passes through the points \((-2, 0)\) and \((0, -4)\). Use these points to find the slope.

\[m = \frac{y_2 - y_1}{x_2 - x_1}\]
\[= \frac{-4 - 0}{0 - (-2)}\]
\[= \frac{-4}{2}\]
\[= -2\]

The \(y\)-intercept is \(-4\).

So, the equation of the line in slope-intercept form is \(y = -2x - 4\).
63. At an amphitheater, the price of 2 lawn seats and 2 pavilion seats is $120. The price of 3 lawn seats and 4 pavilion seats is $225. How much do lawn and pavilion seats cost?

F  $20 and $41.25
G  $10 and $50
H  $15 and $45
J  $30 and $30

SOLUTION:
Let \( l \) = the price of a lawn seat and let \( p \) = the price of a pavilion seat. Then, \( 2l + 2p = 120 \) and \( 3l + 4p = 225 \).

\[
\begin{align*}
2l + 2p &= 120 & \text{Multiply by} \ -2. \\
3l + 4p &= 225 & \text{ } + 2l + 2p = 120 \quad \Rightarrow \quad -4l - 4p = -240 \\
(+)3l + 4p &= 225 & \quad \Rightarrow \quad -l = -15 \\
\quad \Rightarrow \quad l &= 15
\end{align*}
\]

Use the value of \( l \) and either equation to find the value of \( p \).

\[
\begin{align*}
2l + 2p &= 120 \\
2(15) + 2p &= 120 \\
30 + 2p &= 120 \\
\quad + 2p &= 90 \\
\quad 2p &= 90 \\
\quad p &= 45
\end{align*}
\]

The cost of a lawn seat is $15 and the cost of a pavilion seat is $45. So, the correct choice is H.
64. GEOMETRY The circumference of a circle is $\frac{6\pi}{5}$ units. What is the area of the circle?

A $\frac{9\pi}{25}$ units$^2$

B $\frac{3\pi}{5}$ units$^2$

C $\frac{6\pi}{5}$ units$^2$

D $\frac{12\pi}{5}$ units$^2$

**SOLUTION:**

$C = 2\pi r$

$\frac{6\pi}{5} = 2\pi r$

$\frac{6}{5} = 2r$

$\frac{3}{5} = r$

The radius of the circle is $\frac{3}{5}$ units.

$A = \pi r^2$

$= \pi \left(\frac{3}{5}\right)^2$

$= \frac{9}{25} \pi$

The area of the circle is $\frac{9\pi}{25}$ units$^2$. So, the correct choice is A.

**Factor each polynomial, if possible. If the polynomial cannot be factored, write $\text{prime}$.

65. $x^2 - 16$

**SOLUTION:**

$x^2 - 16 = x^2 - 4^2$

$= (x - 4)(x + 4)$

66. $4x^2 - 81y^2$

**SOLUTION:**

$4x^2 - 81y^2 = (2x)^2 - (9y)^2$

$= (2x - 9y)(2x + 9y)$
8-9 Perfect Squares

67. $1 - 100p^2$

**SOLUTION:**

$$1 - 100p^2 = 1^2 - (10p)^2 = (1 - 10p)(1 + 10p)$$

68. $3a^2 - 20$

**SOLUTION:**

The polynomial $3a^2 - 20$ has no common factors or perfect squares. It is prime.

69. $25n^2 - 1$

**SOLUTION:**

$$25n^2 - 1 = (5n)^2 - 1^2 = (5n - 1)(5n + 1)$$

70. $36 - 9c^2$

**SOLUTION:**

$$36 - 9c^2 = 9(4 - c^2) = 9(2 - c)(2 + c)$$
8-9 Perfect Squares

Solve each equation. Confirm your answers using a graphing calculator.
71. $4x^2 - 8x - 32 = 0$

**SOLUTION:**

\[
4x^2 - 8x - 32 = 0 \\
4(x^2 - 2x - 8) = 0 \\
4(x^2 - 4x + 2x - 8) = 0 \\
4[(x^2 - 4x) + (2x - 8)] = 0 \\
4[x(x - 4) + 2(x - 4)] = 0 \\
4(x + 2)(x - 4) = 0
\]

\[x + 2 = 0 \quad \text{or} \quad x - 4 = 0\]
\[x = -2 \quad \text{or} \quad x = 4\]

The roots are −2 and 4.

Confirm the roots using a graphing calculator. Let $Y_1 = 4x^2 - 8x - 32$ and $Y_2 = 0$. Use the **intersect** option from the **CALC** menu to find the points of intersection.

Thus, the solutions are −2 and 4.
8-9 Perfect Squares

72. $6x^2 - 48x + 90 = 0$

**SOLUTION:**

\[
6x^2 - 48x + 90 = 0 \\
6(x^2 - 8x + 15) = 0 \\
6(x^2 - 5x - 3x + 15) = 0 \\
6[x^2 - 5x - (3x - 15)] = 0 \\
6[x(x - 5) - 3(x - 5)] = 0 \\
6(x - 5)(x - 3) = 0
\]

\[
x - 5 = 0 \quad \text{or} \quad x - 3 = 0
\]

\[
x = 5 \\
x = 3
\]

The roots are 3 and 5.

Confirm the roots using a graphing calculator. Let $Y_1 = 6x^2 - 48x + 90$ and $Y_2 = 0$. Use the **intersect** option from the **CALC** menu to find the points of intersection.

Thus, the solutions are 3 and 5.
8-9 Perfect Squares

73. 14x² + 14x = 28

**SOLUTION:**

\[
\begin{align*}
14x^2 + 14x &= 28 \\
14x^2 + 14x - 28 &= 0 \\
14(x^2 + x - 2) &= 0 \\
14(x + 2)(x - 1) &= 0
\end{align*}
\]

\[
x + 2 = 0 \quad \text{or} \quad x - 1 = 0
\]

\[
x = -2 \quad \quad x = 1
\]

The roots are \(-2\) and \(1\).

Confirm the roots using a graphing calculator. Let \(Y_1 = 14x^2 + 14x\) and \(Y_2 = 28\). Use the **intersect** option from the **CALC** menu to find the points of intersection.

Thus, the solutions are \(-2\) and \(1\).
74. $2x^2 - 10x = 48$

**SOLUTION:**

\[
\begin{align*}
2x^2 - 10x &= 48 \\
2x^2 - 10x - 48 &= 0 \\
2(\frac{x^2}{2} - 5x - 24) &= 0 \\
2(\frac{x^2}{2} + 3x - 8x - 24) &= 0 \\
2(\frac{x^2}{2} + 3x) - (8x + 24) &= 0 \\
2\left[\frac{x(x + 3) - 8(x + 3)}{2}\right] &= 0 \\
2(x - 8)(x + 3) &= 0
\end{align*}
\]

\[
\begin{align*}
x - 8 &= 0 \quad \text{or} \quad x + 3 &= 0 \\
x &= 8 \quad \text{or} \quad x &= -3
\end{align*}
\]

The roots are $-3$ and $8$.

Confirm the roots using a graphing calculator. Let $Y_1 = 2x^2 - 10x$ and $Y_2 = 48$. Use the intercept option from the CALC menu to find the points of intersection.

Thus, the solutions are $-3$ and $8$. 

---

**Note:** The diagrams show the points of intersection with $x = -3$ and $x = 8$, confirming the solutions found through algebraic manipulation.
8-9 Perfect Squares

75. \(5x^2 - 25x = -30\)

\textit{SOLUTION:}

\[
\begin{align*}
5x^2 - 25x &= -30 \\
5x^2 - 25x + 30 &= 0 \\
5(\frac{x^2}{5} - 5x + 6) &= 0 \\
5(\frac{x^2}{5} - 2x - 3x + 6) &= 0 \\
5\left[\frac{(x^2 - 2x)}{5} - \frac{(3x - 6)}{5}\right] &= 0 \\
5\left[\frac{x(x - 2)}{5} - \frac{3(x - 2)}{5}\right] &= 0 \\
5\left(\frac{x - 3}{5}\right)\left(\frac{x - 2}{5}\right) &= 0
\end{align*}
\]

\[
\frac{x - 3}{5} = 0 \quad \text{or} \quad \frac{x - 2}{5} = 0
\]

\(
x - 3 = 0 \quad \text{or} \quad x - 2 = 0
\)

\(
x = 3 \quad \text{or} \quad x = 2
\)

The roots are 2 and 3.

Confirm the roots using a graphing calculator. Let \(Y1 = 5x^2 - 25x\) and \(Y2 = -30\). Use the \textbf{intersect} option from the \textbf{CALC} menu to find the points of intersection.

Thus, the solutions are 2 and 3.
76. $8x^2 - 16x = 192$

**SOLUTION:**

\[
\begin{align*}
8x^2 - 16x &= 192 \\
8x^2 - 16x - 192 &= 0 \\
8(x^2 - 2x - 24) &= 0 \\
8(x^2 + 4x - 6x - 24) &= 0 \\
8[(x^2 + 4x) - (6x + 24)] &= 0 \\
8[x(x + 4) - 6(x + 4)] &= 0 \\
8(x - 6)(x + 4) &= 0
\end{align*}
\]

\[
\begin{align*}
x - 6 &= 0 \quad \text{or} \quad x + 4 &= 0 \\
x &= 6 \quad \quad \quad \quad \quad \quad \quad x = -4
\end{align*}
\]

The roots are −4 and 6.

Confirm the roots using a graphing calculator. Let $Y_1 = 8x^2 - 16x$ and $Y_2 = 192$. Use the **intersect** option from the **CALC** menu to find the points of intersection.

Thus, the solutions are −4 and 6.
8-9 Perfect Squares

SOUND The intensity of sound can be measured in watts per square meter. The table gives the watts per square meter for some common sounds.

<table>
<thead>
<tr>
<th>Watts Per Square Meter</th>
<th>Common Sounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-11}$</td>
<td>rustling leaves</td>
</tr>
<tr>
<td>$10^{-10}$</td>
<td>whisper</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>normal conversation</td>
</tr>
<tr>
<td>$10^{-8}$</td>
<td>busy street traffic</td>
</tr>
<tr>
<td>$10^{-7}$</td>
<td>vacuum cleaner</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>front rows of rock concert</td>
</tr>
<tr>
<td>$10^{-5}$</td>
<td>threshold of pain</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>military jet takeoff</td>
</tr>
</tbody>
</table>

77. How many times more intense is the sound from busy street traffic than sound from normal conversation?

**SOLUTION:**
To find how many times more intense the sound from busy street traffic is than sound from normal conversation, divide the watts per square meter for busy street traffic by the watts per square meter for normal conversation.

\[
\frac{10^{-5}}{10^{-6}} = 10^{-(5-(-6))} = 10^{1}
\]

So, the sound from busy street traffic is $10^{1}$ or 10 times more intense that the sound from normal conversation.

78. Which sound is 10,000 times as loud as a busy street traffic?

**SOLUTION:**
To find which sound is 10,000 times as loud as a busy street traffic, multiply the watts per square meter for busy street traffic by 10,000 or $10^{4}$.

\[
10^{-5} \cdot 10^{4} = 10^{-5+4} = 10^{-1}
\]

So, the sound at the front rows of a rock concert is 10,000 times as loud as a busy street traffic.
8-9 Perfect Squares

79. How does the intensity of a whisper compare to that of normal conversation?

**SOLUTION:**
To find how the intensity of a whisper compares to that of normal conversation, divide the watts per meter for a whisper by the watts per square meter for a normal conversation.

\[
\frac{10^{-10}}{10^{-9}} = 10^{-10-(-9)} = 10^{-1} = 10^{-10} \cdot 10^{-(-9)} = 10^{-4} = \frac{1}{10,000}
\]

So, the intensity of a whisper is \(\frac{1}{10,000}\) that of a normal conversation.

**Find the slope of the line that passes through each pair of points.**
80. \((5, 7), (-2, -3)\)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 7}{-2 - 5} = \frac{-10}{-7} = \frac{10}{7}
\]

81. \((2, -1), (5, -3)\)

**SOLUTION:**

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-1)}{5 - 2} = \frac{-2}{3} = -\frac{2}{3}
\]
8.2.\((-4, -1), (-3, -3)\)

**SOLUTION:**
\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-1)}{-3 - (-4)} = \frac{-2}{1} = -2
\]

8.3.\((-3, -4), (5, -1)\)

**SOLUTION:**
\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-4)}{5 - (-3)} = \frac{3}{8}
\]

8.4.\((-2, 3), (8, 3)\)

**SOLUTION:**
\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 3}{8 - (-2)} = 0
\]

8.5.\((-5, 4), (-5, -1)\)

**SOLUTION:**
\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 4}{-5 - (-5)} = \frac{-5}{0} = \text{undefined}
\]